ON THE SIMULTANEOUS SOLVABILITY AND EXPONENTION TO $n+1^{\text{Th}}$ POWER OF PARTICULAR CASE OF QUADRATIC EQUATIONS

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Of the many possible quadratic equations, the paper is concerned about the following type of quadratic equations which can be arbitrarily constructed without the loss of generality:

$$ax^2 + b\sqrt{x} + c = 0 \qquad \S1$$

We will be substituting $y^2 = x$ to make it easier for demonstration as well as grasping the idea.

You are also well aware of the fact that any quadratic has the graph of a parabola. Also parabola has the standard form:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = c \qquad \S2$$

This equation can be constructed with the coefficients in §1 and made be equal to c without loss of generality. Now solve the equation §2 for x^2 to obtain the following generic equation:

$$x^2 = \frac{a^2}{b^2}(b^2c - y^2) \qquad \S3$$

Substitute for x^2 in §1 to arrive at:

$$\frac{a^2}{b^2}(b^2c - y^2).a + by^2 + c = 0 \qquad \S3.\text{A}$$

Also remember that $y^2 = x$ and thus an equivalent form of the result is:

$$\frac{a^3}{b^2}(b^2c - x) + bx + c = 0$$ Or,

$$a^3\left\{c - \frac{x}{b^2}\right\} + bx + c = 0 \qquad \S4$$

Note that now, instead of a complex quadratic, we have an equation in one variable and can be solved using the values of $a, b\ and\ c$ which are all known constants. Also provided you know *x* you can find a suitable relationship between these constants in equations §1 and §2. Now we can further simplify §4 in the following way to obtain a simpler expression for *x* (and indirectly for *y* too):

Now §4 is similar to

$$a^3cb^2 - a^3x + b^3x + b^2c = 0$$

Rearranging the terms:

$$a^3cb^2 + b^2c + b^3x - a^3x = 0$$

Factorizing:

$$b^2c(a^3 + 1) + x(b^3 - a^3) = 0$$

Solving for *x*;

$$x = \frac{-b^2c\left(a^3+1\right)}{(b^3-a^3)} \qquad \S5$$

Squaring both sides:

$$x^2 = \frac{b^4c^2(a^3-1)^2}{(b^3-a^3)^2}$$

Hence *x* can have both positive and negative values:

$$x = \pm\sqrt{\frac{b^4c^2(a^3-1)^2}{(b^3-a^3)^2}}$$ Or,

$$x = \pm\, b^2c.\{\frac{a^3-1}{b^3-a^3}\} \qquad \text{§6}$$

We cannot have the case when the real triplet (a,b,c) that satisfy the equation are all equal

Then, the equation takes the form:

$$a = b = c \leftrightarrow x = \pm a^3\{\frac{a^3-1}{a^3-a^3}\}$$

Hence the conditions for the equations §3 through §6 are:

1. $a, b, c, x, y \in \mathbb{R}$
2. $a \neq b \neq c$ because $\forall\ a = b = c, \nexists\ (a, b, c) \in \mathbb{R}$ (λ)

We can further discover the properties of the equation using another relation;

$$y^2 = \pm 4ax \qquad \text{§7}$$

We have discussed earlier for the case $x = y^2$ a few aspects. Following work approaches §7 for this particular case, keeping in mind (λ).

When $x = y^2$: $x = \pm 4ax = a = \pm\frac{1}{4}$

That is $$y^2 \to x \leftrightarrow a \to \pm\frac{1}{4} \qquad §8$$

Substituting §8 in any of the former equations leads to the desirable results. This concludes the paper.